
FLUID DYNAMICS

*Theory, Computation, and
Numerical Simulation*

Accompanied by the software library *FDLIB*

by

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Preface

Ready access to computers at an institutional and personal level has defined a new era in teaching and learning. The opportunity to extend the subject matter of traditional science and engineering disciplines into the realm of scientific computing has become not only desirable, but also necessary. Thanks to portability and low overhead and operating costs, experimentation by numerical simulation has become a viable substitute, and occasionally the only alternative, to physical experimentation.

The new environment has motivated the writing of texts and monographs with a modern perspective that incorporates numerical and computer programming aspects as an integral part of the curriculum: methods, concepts, and ideas should be presented in a unified fashion that motivates and underlines the urgency of the new elements, but does not compromise the rigor of the classical approach and does not oversimplify.

Interfacing fundamental concepts and practical methods of scientific computing can be done on different levels. In one approach, theory and implementation are kept complementary and presented in a sequential fashion. In a second approach, the coupling involves deriving computational methods and simulation algorithms, and translating equations into computer code instructions immediately following problem formulations. The author of this book is a proponent of the second approach and advocates its adoption as a means of enhancing learning: interjecting methods of scientific computing into the traditional discourse offers a powerful venue for developing analytical skills and obtaining physical insight.

The goal of this book is to offer an introductory course in fluid mechanics, covering traditional topics in a way that unifies theory, computation, computer programming, and numerical simulation. The approach is truly introductory, in the sense that a minimum of prerequisites are required. The intended audience includes not only advanced undergraduate and entry-level graduate students, but also a broad class of scientists and engineers with a general interest in scientific computing.

The discourse is distinguished by two features. First, solution procedures and algorithms are developed immediately after problem formulations. Second, numerical methods are introduced on a need-to-know basis and in increasing order of difficulty: function interpolation, function differentiation, function integration, solution of algebraic equations, finite-difference methods, etc.

A supplement to this book is the FORTRAN software library *FDLIB* whose programs explicitly illustrate how computational algorithms translate into computer code instructions. The codes of *FDLIB* range from introductory to advanced, and the problems considered span a broad range of applications; from laminar channel flows, to vortex flows, to flows in aerodynamics. The input is either entered from the keyboard or read from data files. The output is recorded in output files in numerical form so that it can be read and displayed using independent graphics, visualization, and animation applications on any computer platform. Computer problems at the end of each section ask the student to run the programs for various flow conditions, and thus study the effect of the various parameters characterizing a flow. Instructions for downloading the source code and a description of the library contents are given on page 651.

In concert with the intended usage of this book as a stand-alone text and as a tutorial on numerical fluid dynamics and scientific computing, references are not provided in the text. Instead, a selected compilation of introductory, advanced, and specialized references on fluid dynamics, calculus, numerical methods, and computational fluid dynamics are listed in the bibliography on page 666. The reader who wishes to focus on a particular topic is directed to these resources for further details.

I would like to extend special thanks to Vasilis Bontozoglou for his friendship and encouragement, and to Yuan Chih-Chung, Rhodalynn Degracia, Audrey Hill, and Kurt Keller for helping me with the preparation of the manuscript.

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January, 2001

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FDLIB Software Library

The software library *FDLIB* contains a collection of FORTRAN 77 programs and subroutines that solve a broad range of problems in fluid dynamics using a variety of numerical methods. At the time of this printing, *FDLIB* consist of thirteen main directories, each containing a multitude of nested subdirectories. The contents of the subdirectories are listed on pages 655-667.

Downloading

The source codes of *FDLIB* and accompanying *User Guide*, available in the pdf format, can be downloaded from the internet site:

http://stokes.ucsd.edu/c_pozrikidis/FDLIB

Installation and compilation on UNIX or LINUX

- The library has been archived using the tar UNIX facility into the file *FDLIB.tar*. To unravel the directories on a UNIX or LINUX system, please execute the UNIX command:
`tar xvf FDLIB.tar`
- The downloaded package does not contain object files or executables. An application can be built using the makefile provided in each subdirectory. A makefile is a UNIX script that instructs the operating system how to compile the main program and subroutines, and then link the object files into an executable using an f77 compiler.
- To compile the programs using a FORTRAN 90 compiler, simply make appropriate compiler call substitutions in the makefiles.
- To compile the application named *pindos*, go to the subdirectory where it resides, and type:
`make pindos`
- To remove the object files and output files of the application named *pindos*, go to the subdirectory where it resides, and type:
`make purge`

- To remove the object files, output files, and executable of the application named *pindos*, go to the subdirectory where it resides, and type:
`make clean`

Installation and compilation on Windows and Macintosh

- To unravel the directories on a Windows or Macintosh platform, double-click on the archived tar file and follow the on-screen instructions of the invoked application.
- To compile the link the programs, follow the instructions of your FORTRAN 77 or FORTRAN 90 compiler.

CFDLAB

A subset of *FDLIB* has been combined with the X11 graphics library *vogle* into an integrated application that visualizes the results of the simulations. The source code of *CFDLAB* can be downloaded from the internet site:

http://stokes.ucsd.edu/c_pozrikidis/CFDLAB

FDLIB Directories

	Subject	Directory	Units
1	Numerical methods	<i>01_num_meth</i>	89
2	Grids	<i>02_grids</i>	22
3	Hydrostatics	<i>03_hydrostat</i>	5
4	Various	<i>04_various</i>	27
5	Lubrication	<i>05_lub</i>	4
6	Stokes flow	<i>06_stokes</i>	33
7	Potential flow	<i>07_ptf</i>	25
8	Hydrodynamic stability	<i>08_stab</i>	15
9	Vortex motion	<i>09_vortex</i>	16
10	Boundary layers	<i>10_bl</i>	3
11	Finite difference methods	<i>11_fdm</i>	2
12	Boundary element methods	<i>12_bem</i>	6
13	Turbulence	<i>13_turbo</i>	1

Directory Contents

The thirteen main directories consist of subdirectories that include main programs, assisting subroutines, and utility subroutines. Linked with drivers, the utility subroutines become stand-alone modules; all drivers are provided. A list of the subdirectories with a brief statement of their contents follows. The indicated number of units is the sum of the number of main programs and utility subroutines; assisting subroutines and drivers of utility subroutines are not counted. An extensive description of the problem statement, mathematical formulation, and numerical methods can be found in the *FDLIB User Guide* available from the *FDLIB* internet site.

01_num_meth
*General purpose numerical methods in scientific computing.*¹

<i>Subdirectory</i>	<i>Topic</i>	<i>Units</i>
01_num_comp	General aspects of numerical computation.	14
02_lin_calc	Linear algebra and linear calculus.	11
03_lin_eq	Systems of linear algebraic equations.	10
04_nl_eq	Nonlinear algebraic equations.	8
05_eigen	Eigenvalues and eigenvectors of matrices.	9
06_interp_diff	Function interpolation and differentiation.	8
07_integration	Function integration.	10
08_approximation	Function approximation.	8
09_ode_ivp	Ordinary differential equations; initial-value problems.	1
10_ode_bvp	Ordinary differential equations; boundary value problems.	2
11_pde	Partial differential equations.	5
12_spec_fnc	Computation of special functions.	10

¹This directory accompanies the book: C. Pozrikidis 1998 *Numerical Computation in Science and Engineering*, Oxford University Press.

02_grids

Adaptive discretization, parametrization, representation, and meshing of planar lines, three-dimensional lines, and three-dimensional surfaces.

<i>Subdirectory</i>	<i>Topic</i>	<i>Units</i>
grid_2d	Discretization of a planar line into a graded mesh of straight or circular elements.	3
prd_2d	Adaptive parametrization of planar lines.	5
prd_3d	Adaptive parametrization of three-dimensional lines.	5
prd_ax	Adaptive parametrization of planar lines representing the trace of axisymmetric surfaces in a meridional plane.	3
rec_2d	Interpolation through a rectangular grid.	1
rec_2d_strml	Streamline pattern by interpolation through a rectangular grid.	1
sm_3d_cl_df	Smoothing of a function on a closed surface by surface diffusion.	1
sm_3d_cl_tr	Smoothing of a function on a closed surface by Legendre spectrum truncation.	1
trgl_octa	Triangulation of a closed surface.	1
trgl_octa_hs	Triangulation of an open surface.	1
trgl_sq	Triangulation of a square patch.	1

03_hydrostat
Shapes of interfaces in hydrostatics.

<i>Subdirectory</i>	<i>Topic</i>	<i>Units</i>
drop_2d	Shape of a two-dimensional pendant or sessile drop on a plane.	1
drop_ax	Shape of an axisymmetric pendant or sessile drop on a plane.	1
men_2d	Shape of a two-dimensional meniscus between two parallel plates.	1
men_2d_plate	Shape of a two-dimensional meniscus attached to an inclined plate.	1
men_ax	Shape of an axisymmetric meniscus in a circular tube.	1

04_various
Structure and kinematics of various flows.

<i>Subdirectory</i>	<i>Topic</i>	<i>Units</i>
flow_1d	Steady unidirectional flow in a tube with arbitrary cross section.	1
flow_1d_osc	Oscillatory unidirectional flow in a tube with arbitrary cross section.	1
flow_1d_shear	Unidirectional shear flow over an array of cylinders.	1
spf	Similarity solutions for stagnation-point flows.	1
strml	Streamline patterns of a broad range of flows.	1
strml1	Light version of strml.	1
uni_flow	Steady unidirectional flows with rectilinear or circular streamlines.	15
uni_flow_u	Unsteady unidirectional flows with rectilinear or circular streamlines.	8

05_lub
*Nearly unidirectional lubrication flows
at low Reynolds numbers.*

<i>Subdirectory</i>	<i>Topic</i>	<i>Units</i>
bear_2d	Dynamical simulation of the motion of a slider bearing pressing against a wall.	1
chan_2l_exp	Dynamical simulation of the evolution of two superposed viscous layers in a horizontal or inclined channel, computed by an explicit finite-difference method.	1
chan_2l_imp	Dynamical simulation of the evolution of two superposed viscous layers in a horizontal or inclined channel, computed by an implicit finite-difference method.	1
films	Evolution of an arbitrary number of superposed films on a horizontal or plane wall.	1

*06_stokes**Viscous flows at vanishing Reynolds numbers.*

<i>Subdirectory</i>	<i>Topic</i>	<i>Units</i>
caps_2d	Dynamical simulation of the motion of a two-dimensional drop or elastic capsule, for a variety of flow configurations.	1
caps_3d	Dynamical simulation of the motion of a three-dimensional elastic capsule.	1
caps_ax	Dynamical simulation of the motion of an axisymmetric drop or elastic capsule, for a variety of flow configurations.	1
cop_ax	Shear flow over an axisymmetric cavity, orifice, or protrusion.	1
drop_3d	Dynamical simulation of the motion of a three-dimensional drop with constant or varying surface tension.	1
drop_3dw	Dynamical simulation of the deformation of a three-dimensional drop adhering to a plane wall.	1
em_2d	Dynamical simulation of the motion of a suspension of two-dimensional drops or elastic capsules, for a variety of flow configurations.	1
films	Dynamical simulation of the motion of superimposed layers in a channel, or two films flowing down a plane wall.	1
flow_2d	Two-dimensional flow in a domain with arbitrary geometry.	1
layers	Dynamical simulation of the motion of an arbitrary number of layers in a channel, or films flowing down a plane wall.	1
prtcl_2d	Flow past a fixed bed of two-dimensional particles with arbitrary shapes, for a variety of flow configurations, computed by a boundary-element method.	1

*06_stokes (Continued)**Viscous flows at vanishing Reynolds numbers.*

<i>Subdirectory</i>	<i>Topic</i>	<i>Units</i>
prtcl_2d_se	Flow past a fixed bed of two-dimensional particles with arbitrary shapes for a variety of flow configurations, computed by a spectral-element method.	1
prtcl_3d	Flow past, or due to the motion of, a three-dimensional particle, for a variety of configurations, computed by a boundary-element method.	1
prtcl_ax	Flow past, or due to the motion of, a collection of axisymmetric particles, computed by a boundary-element method.	1
prtcl_sw	Swirling flow produced by the rotation of an axisymmetric particle, computed by a boundary-element method.	1
sgf_2d	Green's functions of two-dimensional Stokes flow.	6
sgf_3d	Green's functions of three-dimensional Stokes flow.	5
sgf_3dax	Green's functions of Stokes flow in an axisymmetric domain.	1
sgf_ax	Green's functions of axisymmetric Stokes flow.	4
susp_2d	Dynamical simulation of the motion of a suspension of two-dimensional rigid particles with arbitrary shapes, for a variety of flow configurations, computed by a boundary-element method.	1
susp_2d_se	Dynamical simulation of the motion of a suspension of two-dimensional rigid particles with arbitrary shapes, for a variety of flow configurations, computed by a spectral-element method.	1
thread_ax	Dynamical simulation of the evolution of a fluid thread or annular layer.	1

07_ptf

Potential flows.

<i>Subdirectory</i>	<i>Topic</i>	<i>Units</i>
airf_2d	Shapes of airfoils.	1
airf_2d.cdp	Flow past an airfoil computed by the constant-dipole-panel method.	1
airf_2d.csdp	Flow past an airfoil computed by the constant-source-dipole-panel method.	1
airf_2d.lvp	Flow past an airfoil computed by the linear-vortex-panel method.	
body_2d	Flow past, or due to the motion of, a two-dimensional body, computed by a boundary element method.	1
body_ax	Flow past, or due to the motion of, an axisymmetric body, computed by a boundary element method.	
bubble_3d	Dynamical simulation of the deformation, collapse, or oscillations of a three-dimensional bubble.	1
cvt_2d	Flow in a rectangular cavity, computed by a finite difference method.	1
drop_3d	Dynamical simulation of the surface-tension induced oscillations of a three-dimensional inviscid drop suspended in vacuum.	1
flow_2d	Two-dimensional flow in an arbitrary domain, computed by a boundary element method.	1
lgf_2d	Green and Neumann functions of Laplace's equation in two dimensions.	8
lgf_3d	Green and Neumann functions of Laplace's equation in three dimensions.	5
lgf_ax	Green and Neumann functions of Laplace's equation in axisymmetric domains.	3
tank_2d	Dynamical simulation of liquid sloshing in a rectangular tank, computed by a boundary integral method.	1