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Screw Compressors

Mathematical Modelling and Performance Calculation

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With 99 Figures

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Library of Congress Control Number: 2004117305

ISBN-10 3-540-24275-9 Springer Berlin Heidelberg New York
ISBN-13 978-3-540-24275-8 Springer Berlin Heidelberg New York

Springer is a part of Springer Science+Business Media
springeronline.com

Printed in The Netherlands

Typesetting: by the authors and TechBooks using a Springer LaTeX macro package
Cover design: medio, Berlin

Printed on acid-free paper SPIN: 11306856 62/3141/jl 5 4 3 2 1 0

Preface

Although the principles of operation of helical screw machines, as compressors or expanders, have been well known for more than 100 years, it is only during the past 30 years that these machines have become widely used. The main reasons for the long period before they were adopted were their relatively poor efficiency and the high cost of manufacturing their rotors. Two main developments led to a solution to these difficulties. The first of these was the introduction of the asymmetric rotor profile in 1973. This reduced the blowhole area, which was the main source of internal leakage by approximately 90%, and thereby raised the thermodynamic efficiency of these machines, to roughly the same level as that of traditional reciprocating compressors. The second was the introduction of precise thread milling machine tools at approximately the same time. This made it possible to manufacture items of complex shape, such as the rotors, both accurately and cheaply.

From then on, as a result of their ever improving efficiencies, high reliability and compact form, screw compressors have taken an increasing share of the compressor market, especially in the fields of compressed air production, and refrigeration and air conditioning, and today, a substantial proportion of compressors manufactured for industry are of this type.

Despite, the now wide usage of screw compressors and the publication of many scientific papers on their development, only a handful of textbooks have been published to date, which give a rigorous exposition of the principles of their operation and none of these are in English.

The publication of this volume coincides with the tenth anniversary of the establishment of the Centre for Positive Displacement Compressor Technology at City University, London, where much, if not all, of the material it contains was developed. Its aim is to give an up to date summary of the state of the art. Its availability in a single volume should then help engineers in industry to replace design procedures based on the simple assumptions of the compression of a fixed mass of ideal gas, by more up to date methods. These are based on computer models, which simulate real compression and expansion processes more reliably, by allowing for leakage, inlet and outlet flow and other losses, and the assumption of real fluid properties in the working process. Also, methods are given for developing rotor profiles, based on the mathematical theory of gearing, rather than empirical curve fitting. In addition, some description is included of procedures for the three dimensional modelling of heat and fluid flow through these machines and how interaction between the rotors and the casing produces performance changes, which hitherto could not be calculated. It is shown that only a relatively small number of input parameters is required to describe both the geometry and performance of screw compressors. This makes it easy to control the design process so that modifications can be cross referenced through design software programs, thus saving both computer resources and design time, when compared with traditional design procedures.

All the analytical procedures described, have been tried and proven on machines currently in industrial production and have led to improvements in performance and reductions in size and cost, which were hardly considered possible ten years ago. Moreover, in all cases where these were applied, the improved accuracy of the analytical models has led to close agreement between predicted and measured performance which greatly reduced development time and cost. Additionally, the better understanding of the principles of operation brought about by such studies has led to an extension of the areas of application of screw compressors and expanders.

It is hoped that this work will stimulate further interest in an area, where, though much progress has been made, significant advances are still possible.

London,
February 2005

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Notation

A	Area of passage cross section, oil droplet total surface
a	Speed of sound
C	Rotor centre distance, specific heat capacity, turbulence model constants
d	Oil droplet Sauter mean diameter
e	Internal energy
$\mathbf{f}$	Body force
h	Specific enthalpy $h = h(\theta)$, convective heat transfer coefficient between oil and gas
$\mathbf{i}$	Unit vector
$\mathbf{I}$	Unit tensor
k	Conductivity, kinetic energy of turbulence, time constant
m	Mass
$\dot{m}$	Inlet or exit mass flow rate $\dot{m} = \dot{m}(\theta)$
p	Rotor lead, pressure in the working chamber $p = p(\theta)$
P	Production of kinetic energy of turbulence
$\mathbf{q}$	Source term
$\dot{Q}$	Heat transfer rate between the fluid and the compressor surroundings $\dot{Q} = \dot{Q}(\theta)$
r	Rotor radius
s	Distance between the pole and rotor contact points, control volume surface
t	Time
T	Torque, Temperature
$\mathbf{u}$	Displacement of solid
U	Internal energy
W	Work output
$\mathbf{v}$	Velocity
$\mathbf{w}$	Fluid velocity
V	Local volume of the compressor working chamber $V = V(\theta)$
$\dot{V}$	Volume flow

x Rotor coordinate, dryness fraction, spatial coordinate
y Rotor coordinate
z Axial coordinate

Greek Letters

α Temperature dilatation coefficient
Γ Diffusion coefficient
ε Dissipation of kinetic energy of turbulence
η_i Adiabatic efficiency
η_t Isothermal efficiency
η_v Volumetric efficiency
φ Specific variable
ϕ Variable
λ Lame coefficient
μ Viscosity
ρ Density
σ Prandtl number
θ Rotor angle of rotation
ζ Compound, local and point resistance coefficient
ω Angular speed of rotation

Prefixes

d differential
Δ Increment

Subscripts

eff Effective
g Gas
in Inflow
f Saturated liquid
g Saturated vapour
ind Indicator
l Leakage
oil Oil
out Outflow
p Previous step in iterative calculation
s Solid
T Turbulent
w pitch circle
1 main rotor, upstream condition
2 gate rotor, downstream condition

Contents

1

Introduction

The screw compressor is one of the most common types of machine used to compress gases. Its construction is simple in that it essentially comprises only a pair of meshing rotors, with helical grooves machined in them, contained in a casing, which fits closely round them. The rotors and casing are separated by very small clearances. The rotors are driven by an external motor and mesh like gears in such a manner that, as they rotate, the space formed between them and the casing is reduced progressively. Thus, any gas trapped in this case is compressed. The geometry of such machines is complex and the flow of the gas being compressed within them occurs in three stages. Firstly, gas enters between the lobes, through an inlet port at one end of the casing during the start of rotation. As rotation continues, the space between the rotors no longer lines up with the inlet port and the gas is trapped and thus compressed. Finally, after further rotation, the opposite ends of the rotors pass a second port at the other end of the casing, through which the gas is discharged. The whole process is repeated between successive pairs of lobes to create a continuous but pulsating flow of gas from low to high pressure.

These machines are mainly used for the supply of compressed air in the building industry, the food, process and pharmaceutical industries and, where required, in the metallurgical industry and for pneumatic transport. They are also used extensively for compression of refrigerants in refrigeration and air conditioning systems and of hydrocarbon gases in the chemical industry. Their relatively rapid acceptance over the past thirty years is due to their relatively high rotational speeds compared to other types of positive displacement machine, which makes them compact, their ability to maintain high efficiencies over a wide range of operating pressures and flow rates and their long service life and high reliability. Consequently, they constitute a substantial percentage of all positive displacement compressors now sold and currently in operation.

The main reasons for this success are the development of novel rotor profiles, which have drastically reduced internal leakage, and advanced machine tools, which can manufacture the most complex shapes to tolerances of the order of 3 micrometers at an acceptable cost. Rotor profile enhancement is still the most promising means of further improving screw compressors and rational procedures are now being developed both to replace earlier empirically derived shapes and also to vary the proportions of the selected profile to obtain the best result for the application for which the compressor is required. Despite their wide usage, due to the complexity of their internal geometry and the non-steady nature of the processes within them, up till recently, only approximate analytical methods have been available to predict their performance. Thus, although it is known that their elements are distorted both by the heavy loads imposed by pressure induced forces and through temperature changes within them, no methods were available to predict the magnitude of these distortions accurately, nor how they affect the overall performance of the machine. In addition, improved modelling of flow patterns within the machine can lead to better porting design. Also, more accurate determination of bearing loads and how they fluctuate enable better choices of bearings to be made. Finally, if rotor and casing distortion, as a result of temperature and pressure changes within the compressor, can be estimated reliably, machining procedures can be devised to minimise their adverse effects.

Screw machines operate on a variety of working fluids, which may be gases, dry vapour or multi-phase mixtures with phase changes taking place within the machine. They may involve oil flooding, or other fluids injected during the compression or expansion process, or be without any form of internal lubrication. Their geometry may vary depending on the number of lobes in each rotor, the basic rotor profile and the relative proportions of each rotor lobe segment. It follows that there is no universal configuration which would be the best for all applications. Hence, detailed thermodynamic analysis of the compression process and evaluation of the influence of the various design parameters on performance is more important to obtain the best results from these machines than from other types which could be used for the same application. A set of well defined criteria governed by an optimisation procedure is therefore a prerequisite for achieving the best design for each application. Such guidelines are also essential for the further improvement of existing screw machine designs and broadening their range of uses. Fleming et al., 1998 gives a good contemporary review of screw compressor modelling, design and application.

A mathematical model of the thermodynamic and fluid flow processes within positive displacement machines, which is valid for both the screw compressor and expander modes of operation, is presented in this Monograph. It includes the use of the equations of conservation of mass, momentum and energy applied to an instantaneous control volume of trapped fluid within the machine with allowance for fluid leakage, oil or other fluid injection, heat transfer and the assumption of real fluid properties. By simultaneous solution of these equations, pressure-volume diagrams may be derived of the entire admission, discharge and compression or expansion process within the machine.

A screw machine volume is defined by the rotor profile which is here generated by use of a general gearing algorithm and the port shape and size. This algorithm demonstrates the meshing condition which, when solved explicitly, enables a variety of rotor primary arcs to be defined either analytically or by discrete point curves. Its use greatly simplifies the design since only primary arcs need to be specified and these can be located on either the main or gate rotor or even on any other rotor including a rack, which is a rotor of infinite radius. The most efficient profiles have been obtained from a combined rotor-rack generation procedure.

The rotor profile generation processor, thermofluid solver and optimizer, together with pre-processing facilities for the input data and graphical post processing and CAD interface, have been incorporated into a design tool in the form of a general computer code which provides a suitable tool for analysis and optimization of the lobe profiles and other geometrical and physical parameters. The Monograph outlines the adopted rationale and method of modelling, compares the shapes of the new and conventional profiles and illustrates potential improvements achieved with the new design when applied to dry and oil-flooded air compressors as well as to refrigeration screw compressors.

The first part of the Monograph gives a review of recent developments in screw compressors.

The second part presents the method of mathematical definition of the general case of screw machine rotors and describes the details of lobe shape specification. It focuses on a new lobe profile of a slender shape with thinner lobes in the main rotor, which yields a larger cross-sectional area and shorter sealing lines resulting in higher delivery rates for the same tip speed.

The third part describes a model of the thermodynamics of the compression-expansion processes, discusses some modelling issues and compares the shapes of new and conventional profiles. It illustrates the potential improvements achievable with the new design applied to dry and oil-flooded air compressors as well as to refrigeration screw compressors. The selection of the best gate rotor tip radius is given as an example of how mathematical modelling may be used to optimise the design and the machine's operating conditions.

The fourth part describes the design of a high efficiency screw compressor with new rotor profiles. A well proven mathematical model of the compression process within positive displacement machines was used to determine the optimum rotor size and speed, the volume ratio and the oil injection position and jet diameter. In addition, modern design concepts such as an open suction port and early exposure of the discharge port were included, together with improved bearing and seal specification, to maximise the compressor efficiency. The prototypes were tested and compared with the best compressors currently on the market. The measured specific power input appeared to be lower than any published values for other equivalent compressors currently manufactured. Both the predicted advantages of the new rotor profile and the superiority of the design procedure were thereby confirmed.

1.1 Basic Concepts

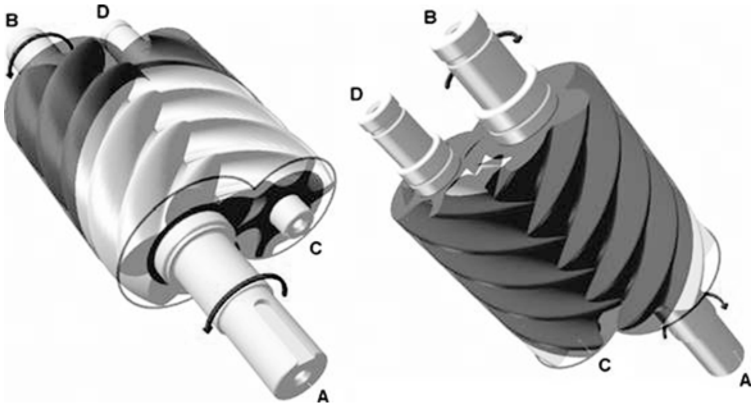
Thermodynamic machines for the compression and expansion of gases and vapours are the key components of the vast majority of power generation and refrigeration systems and essential for the production of compressed air and gases needed by industry. Such machines can be broadly classified by their mode of operation as either turbomachines or those of the positive displacement type.

Turbomachines effect pressure changes mainly by dynamic effects, related to the change of momentum imparted to the fluids passing through them. These are associated with the steady flow of fluids at high velocities and hence these machines are compact and best suited for relatively large mass flow rates. Thus compressors and turbines of this type are mainly used in the power generation industry, where, as a result of huge investment in research and development programmes, they are designed and built to attain thermodynamic efficiencies of more than 90% in large scale power production plant. However, the production rate of machines of this type is relatively small and worldwide, is only of the order of some tens of thousands of units per annum.

Positive displacement machines effect pressure changes by admitting a fixed mass of fluid into a working chamber where it is confined and then compressed or expanded and, from which it is finally discharged. Such machines must operate more or less intermittently. Such intermittent operation is relatively slow and hence these machines are comparatively large. They are therefore better suited for smaller mass flow rates and power inputs and outputs. A number of types of machine operate on this principle such as reciprocating, vane, scroll and rotary piston machines.

In general, positive displacement machines have a wide range of application, particularly in the fields of refrigeration and compressed air production and their total world production rate is in excess of 200 million units per annum. Paradoxically, but possibly because these machines are produced by comparatively small companies with limited resources, relatively little is spent on research and development programmes on them and there are very few academic institutions in the world which are actively promoting their improvement.

One of the most successful positive displacement machines currently in use is the screw or twin screw compressor. Its principle of operation, as indicated in Fig. 1.1, is based on volumetric changes in three dimensions rather than two. As shown, it consists, essentially, of a pair of meshing helical lobed rotors, contained in a casing. The spaces formed between the lobes on each rotor form a series of working chambers in which gas or vapour is contained. Beginning at the top and in front of the rotors, shown in the light shaded portion of Fig. 1.1a, there is a starting point for each chamber where the trapped volume is initially zero. As rotation proceeds in the direction of the arrows, the volume of that chamber then increases as the line of contact between the rotor with convex lobes, known as the main rotor, and the adjacent lobe of the gate rotor



(a) View from Front and Top (b) View from Bottom and Rear

Fig. 1.1. Screw Compressor Rotors

advances along the axis of the rotors towards the rear. On completion of one revolution i.e. 360° by the main rotor, the volume of the chamber is then a maximum and extends in helical form along virtually the entire length of the rotor. Further rotation then leads to reengagement of the main lobe with the succeeding gate lobe by a line of contact starting at the bottom and front of the rotors and advancing to the rear, as shown in the dark shaded portions in Fig. 1.1b. Thus, the trapped volume starts to decrease. On completion of a further 360° of rotation by the main rotor, the trapped volume returns to zero.

The dark shaded portions in Fig. 1.1 show the enclosed region where the rotors are surrounded by the casing, which fits closely round them, while the light shaded areas show the regions of the rotors, which are exposed to external pressure. Thus the large light shaded area in Fig. 1.1a corresponds to the low pressure port while the small light shaded region between shaft ends B and D in Fig. 1.1b corresponds to the high pressure port.

Exposure of the space between the rotor lobes to the suction port, as their front ends pass across it, allows the gas to fill the passages formed between them and the casing until the trapped volume is a maximum. Further rotation then leads to cut off of the chamber from the port and progressive reduction in the trapped volume. This leads to axial and bending forces on the rotors and also to contact forces between the rotor lobes. The compression process continues until the required pressure is reached when the rear ends of the passages are exposed to the discharge port through which the gas flows out at approximately constant pressure.

It can be appreciated from examination of Fig. 1.1, is that if the direction of rotation of the rotors is reversed, then gas will flow into the machine through the high pressure port and out through the low pressure port and it will act as an expander. The machine will also work as an expander when rotating in the same direction as a compressor provided that the suction and discharge ports are positioned on the opposite sides of the casing to those shown since this is effectively the same as reversing the direction of rotation relative to the ports. When operating as a compressor, mechanical power must be supplied to shaft A to rotate the machine. When acting as an expander, it will rotate automatically and power generated within it will be supplied externally through shaft A.

The meshing action of the lobes, as they rotate, is the same as that of helical gears but, in addition, their shape must be such that at any contact position, a sealing line is formed between the rotors and between the rotors and the casing in order to prevent internal leakage between successive trapped passages. A further requirement is that the passages between the lobes should be as large as possible, in order to maximise the fluid displacement per revolution. Also, the contact forces between the rotors should be low in order to minimise internal friction losses. A typical screw rotor profile is shown in Fig. 1.2, where a configuration of 5–6 lobes on the main and gate rotors is presented. The meshing rotors are shown with their sealing lines, for the axial plane on the left and for the cross-sectional plane in the centre. Also, the clearance distribution between the two rotor racks in the transverse plane, scaled 50 times (6) is given above.

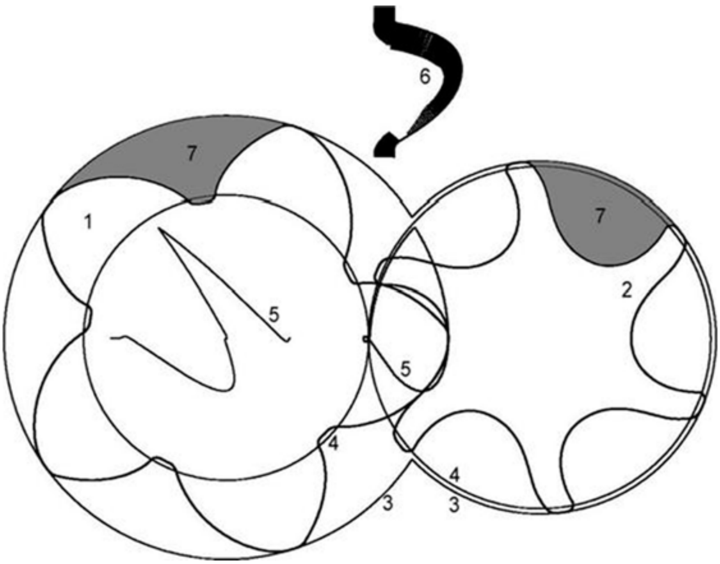


Fig. 1.2. Screw rotor profile: (1) main, (2) gate, (3) rotor external and (4) pitch circles, (5) sealing line, (6) clearance distribution and (7) rotor flow area between the rotors and housing

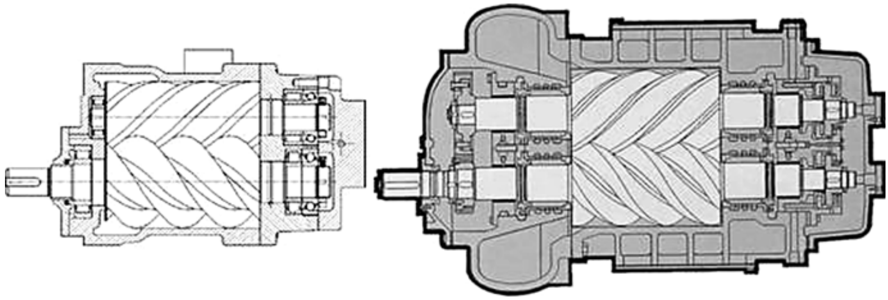


Fig. 1.3. Oil Injected and Oil Free Compressors

Screw machines have a number of advantages over other positive displacement types. Firstly, unlike reciprocating machines, the moving parts all rotate and hence can run at much higher speeds. Secondly, unlike vane machines, the contact forces within them are low, which makes them very reliable. Thirdly, and far less well appreciated, unlike the reciprocating, scroll and vane machines, all the sealing lines of contact which define the boundaries of each cell chamber, decrease in length as the size of the working chamber decreases and the pressure within it rises. This minimises the escape of gas from the chamber due to leakage during the compression or expansion process.

1.2 Types of Screw Compressors

Screw compressors may be broadly classified into two types. These are shown in Fig. 1.3 where machines with the same size rotors are compared:

1.2.1 The Oil Injected Machine

This relies on relatively large masses of oil injected with the compressed gas in order to lubricate the rotor motion, seal the gaps and reduce the temperature rise during compression. It requires no internal seals, is simple in mechanical design, cheap to manufacture and highly efficient. Consequently it is widely used as a compressor in both the compressed air and refrigeration industries.

1.2.2 The Oil Free Machine

Here, there is no mixing of the working fluid with oil and contact between the rotors is prevented by timing gears which mesh outside the working chamber and are lubricated externally. In addition, to prevent lubricant entering the working chamber, internal seals are required on each shaft between the working chamber and the bearings. In the case of process gas compressors, double mechanical seals are used. Even with elaborate and costly systems such as these, successful internal sealing is still regarded as a problem by established process gas compressor manufacturers. It follows that such machines are considerably more expensive to manufacture than those that are oil injected.

Both types require an external heat exchanger to cool the lubricating oil before it is readmitted to the compressor. The oil free machine requires an oil tank, filters and a pump to return the oil to the bearings and timing gear.

The oil injected machine requires a separator to remove the oil from the high pressure discharged gas but relies on the pressure difference between suction and discharge to return the separated oil to the compressor. These additional components increase the total cost of both types of machine but the add on cost is greater for the oil free compressor.

1.3 Screw Machine Design

Serious efforts to develop screw machines began in the nineteen thirties, when turbomachines were relatively inefficient. At that time, Alf Lysholm, a talented Swedish engineer, required a high speed compressor, which could be coupled directly to a turbine to form a compact prime mover, in which the motion of all moving parts was purely rotational. The screw compressor appeared to him to be the most promising device for this purpose and all modern developments in these machines stem from his pioneering work. Typical screw compressor designs are presented in Figs. 1.4 and 1.5. From then until the mid nineteen sixties, the main drawback to their widespread use was the inability to manufacture rotors accurately at an acceptable cost. Two developments then accelerated their adoption. The first was the development of milling machines for thread cutting. Their use for rotor manufacture enabled these components to be made far more accurately at an acceptable cost. The second occurred in nineteen seventy three, when SRM, in Sweden, introduced the "A" profile, which reduced the internal leakage path area, known as the blow hole, by 90%. Screw compressors could then be built with efficiencies approximately equal to those of reciprocating machines and, in their oil flooded form, could operate efficiently with stage pressure ratios of up to 8:1. This was unattainable with reciprocating machines. The use of screw compressors, especially of the oil flooded type, then proliferated.

To perform effectively, screw compressor rotors must meet the meshing requirements of gears while maintaining a seal along their length to minimise leakage at any position on the band of rotor contact. It follows that the compressor efficiency depends on both the rotor profile and the clearances between the rotors and between the rotors and the compressor housing.

Screw compressor rotors are usually manufactured on specialised machines by the use of formed milling or grinding tools. Machining accuracy achievable today is high and tolerances in rotor manufacture are of the order of 5 μm around the rotor lobes. Holmes, 1999 reported that even higher accuracy was

Fig. 1.4. Screw compressor mechanical parts

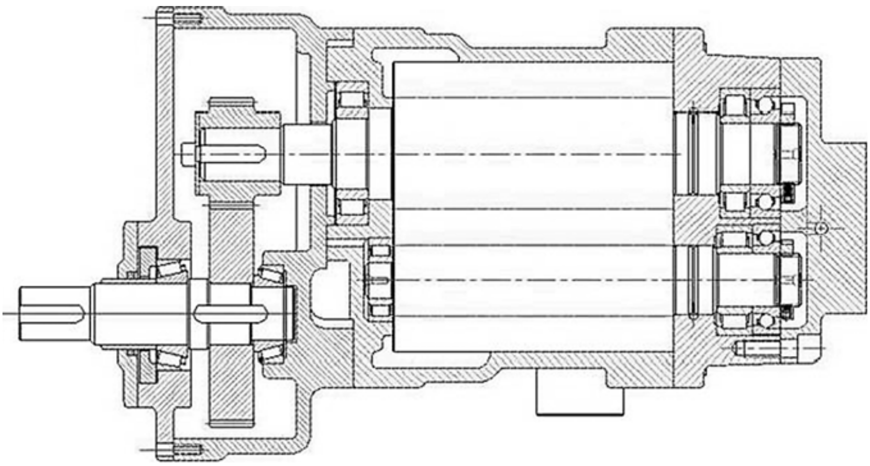


Fig. 1.5. Cross section of a screw compressor with gear box

achieved on the new Holroyd vitrifying thread-grinding machine, thus keeping the manufacturing tolerances within 3 µm even in large batch production. This means that, as far as rotor production alone is concerned, clearances between the rotors can be as small as 12 µm.

Screw machines are used today for different applications both as compressors and expanders. For optimum performance from them a specific design