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Computability and Models

Perspectives East and West

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Preface

Science involves *descriptions* of the world we live in. It also depends on nature exhibiting what we can best describe as a high *algorithmic content*. The theme running through this collection of papers is that of the interaction between descriptions, in the form of formal theories, and the algorithmic content of what is described, namely of the *models* of those theories. This appears most explicitly here in a number of valuable, and substantial, contributions to what has until recently been known as 'recursive model theory' – an area in which researchers from the former Soviet Union (in particular Novosibirsk) have been pre-eminent. There are also articles concerned with the computability of aspects of familiar mathematical structures, and — a return to the sort of basic underlying questions considered by Alan Turing in the early days of the subject — an article giving a new perspective on computability in the real world. And, of course, there are also articles concerned with the classical theory of computability, including the first widely available survey of work on quasi-reducibility.

The contributors, all internationally recognised experts in their fields, have been associated with the three-year INTAS-RFBR Research Project "Computability and Models" (Project No. 972-139), and most have participated in one or more of the various international workshops (in Novosibirsk, Heidelberg and Almaty) and other research activities of the network. Although based on just eight research centres – Almaty, Heidelberg, Ivanovo, Kazan, Leeds, Novosibirsk, Siena and Turin – the project has acted as a focus for researchers from all over Europe and beyond. This has been an exciting and rewarding experience for everybody involved, and has helped transform the fragmented European scene of ten or more years ago (so vividly described by George Odifreddi in his entertaining introduction to this volume) into the lively community of researchers we now see developing.

The articles which follow approach this important and growing area of research from many different angles. The authors were encouraged to provide *readable* introductions to their research. All have responded either with timely surveys of work inadequately covered elsewhere, or with interesting and important new results, with clear pointers to the wider context. All articles have been rigorously refereed, and revised accordingly. We wish to express our gratitude to the contributors, and hope that the reader will be able to benefit from the expertise and enthusiasm expressed in their articles.

We also wish to acknowledge with thanks the support given by the EU, through INTAS, and by the RFBR, without which this volume would not have been possible.

BARRY COOPER

SERGEY GONCHAROV

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Piergiorgio Odifreddi

Introduction

"East is East and West is West, and never the twain shall meet"

says Rudyard Kipling in *The Ballad of East and West.* It certainly looked so to most recursion theorists in the late seventies, except for a vague knowledge of the fact that Post's Problem had been solved independently by the Soviet mathematician Muchnik in 1956, and the North American Richard Friedberg in 1957.

In his book, *Theory of Recursive Functions and Effective Computability,* Hartley Rogers called the result "the Friedberg-Muchnik Theorem", ironically preferring the alphabetical to the priority order! Except for the references to this theorem, Muchnik is mentioned only once (in an exercise) by Rogers, and never by Bob Soare in *Recursively Enumerable Sets and Degrees,* despite the fact that he had obtained a number of other results: first and foremost, what everybody (except me) still calls "the Friedberg Splitting Theorem".

From Rogers' book one got the impression that Recursion Theory was a totally Western enterprise, with practically no Eastern contribution besides Muchnick's. At least, this was the impression I had formed when I landed in Urbana in 1978, to study with Carl Jockusch. An impression that was to change very quickly, thanks to him and Marat Arslanov, who had arrived a few days earlier than I. He was met at the airport by Carl who, after unsuccessfully trying to communicate with him in English or German, asked whether he spoke any language besides Russian, and got the answer: "Oh yes! Tatar."

Jockusch was the foremost expert on what are now called "strong reducibilities", and knew that much work had been done in the Soviet Union. In particular, he was aware of Marchenkov's new solution to Post's Problem and Ershov's characterization of the structure of the m-degrees. However, the original sources were not easy to handle, because of a two-fold problem: they were either Russian originals, or English translations of abstracts without proofs.

During the year I spent in Urbana I wrote a long paper on "Strong Reducibilities", with a lot of input from Carl. I handed him my only copy, handwritten in pencil. After returning the first half of it with his comments, he suggested that I expand the treatment to other strong reducibilities. When I asked him whether the second part was not enough, he asked surprised: "Which second part?" For a few days we thought we would never see it again, but it eventually re-emerged from his maximum entropy desk in the very colourful office he was sharing with Ken Appel. The latter had just proved his big theorem, and the University was happily stamping all its mail with: "Four Colours Suffice!". By coincidence Ken had the same birthday as Carl and I, and that year we all celebrated together.

As for my paper, I sent it to the *Bulletin of the American Mathematical Society*, without knowing that the New Series surveys were not supposed to be submitted by postdocs, but rather requested from luminaries. Luckily, however, the paper was accepted, on condition that a general introduction explained the purposes of Recursion Theory and its relationships to the rest of mathematics. This was done "with a little help from my friends", Richard Shore and Bob Soare in particular, and the publication earned me an unexpected check of a thousand dollars from the American Mathematical Society, for reasons that I did not attempt to clarify (fearing a mistake). Less personally, the paper also played a certain role in making some Eastern work known to a Western audience.

That was the most I could do from a distance. To proceed further it was necessary to go to the Soviet Union, which I did in 1982, after having studied Russian (and other things, of course) in Los Angeles for a year. My flight from Moscow to Novosibirsk was forced to stop in Omsk because of a snow storm, and I was taken to some hotel by Inturist. A couple of hours later, since the weather had improved, I was thrown out of bed and taken to my destination. Obviously, nobody was expecting me anymore and I had to wait for the rest of the night sitting on an airport chair. In the morning Serghei Goncharov came to pick me up, introduced himself and asked me how I had slept. I did not dare start off with a complaint, and as a result I was immediately whisked to the weekly seminar. After the greetings of Yuri Ershov, whom I had already met in Los Angeles, the seminar started and I immediately and happily fell asleep.

The seminar in Novosibirsk was to logicians what Picadilly Circus is to tourists: if you want to meet someone, you just wait there and sooner or later he will come. I thus met most of the Soviet recursion theorists whose work I had quoted in "Strong Reducibilities": in particular Alexander Degtev, Sergei Denisov, and Victor Selivanov, who explained to me their results and helped me translate their "Eastern-style" proofs into "Western-style" language.

This was not always easy or straightforward, because they usually exploited structural properties of the r.e. sets to derive degree-theoretical ones, instead of using direct constructions. Their approach was more informative but less flexible, and they did miss a number of results because of it. A typical case is Degtev's structural proof of the existence of a singular r.e. tt-degree. When Rod Downey found a direct construction, he was able to adapt it to derive a wealth of theorems on the structure of r.e. m-degrees inside r.e. tt-degrees.

Some of the Soviets have later been converted, at least partially, to our way of working. For example, Selivanov once flattered me by saying that when he saw my treatment of his structural results on the m-degrees of index sets he thought: "Finally I understand what I did!" And Arslanov published a book on degrees which followed closely, at times perhaps even too much, the first (and otherwise unpublished) draft of *Classical Recursion Theory*.

The greatest difficulty I encountered was the "translation" of Ershov's characterization of the structure of the m-degrees, which he had obtained in the framework of his Theory of Enumerations. I had to develop everything from scratch, and the result of this work is now Chapter VI of my book. Once again, the direct proof could be adapted to derive similar results in other settings, in particular for the polynomial time honest m-degrees.

Incidentally, although this had no effect on my work, one of the mathematicians I met in Novosibirsk was Efim Zelmanov. He was then a brilliant young Jew, who at the time of my visit was not allowed to attend the 1983 Warsaw International Congress of Mathematicians and give one of the invited talks. He did attend the 1994 Zürich Congress, where he was awarded a Fields Medal for his solution of a restricted version of the first Burnside conjecture in group theory.

In Novosibirsk I saw again Arslanov, whom I also visited in Kazan. While in Urbana we could not speak at ease, because none of us was comfortable with English, but this time I had learned enough Russian to appreciate his wonderful sense of humour. For example, we were once asked which of the definitions that we had both given of "effective hypersimplicity" was better, and he immediately answered: "George's is more general, but mine is more important". The structure of differences of r.e. sets, whose study he had independently initiated with his students Bukaraev and Ishmuchametov, is now a favourite in the West.

In Moscow I made a point of meeting Sergei Marchenkov, who treated me very nicely and offered me a wonderful lunch, but refused to talk about Recursion Theory. At the time I could not understand how he could have lost interest in such a beautiful field, to the point of not even remembering his own results, but I'm afraid time has taught me the lesson. I did not meet Muchnik, probably because he was away, but I remember a surreal visit to Vladimir Ouspensky. He decided the priority order in which we all had to enter and leave the room, and greeted me with a: "I hope you're not a shitty set theorist!" When I asked him what was "shitty" in Set Theory, he told me that it was a perfect image of capitalism: you start with nothing, and accumulate by building on power ...

In Tbilisi I was hosted by Georgi Kobzev and Roland Omanadze, who told me of their surprise in opening one day the *Bulletin of the American Mathematical Society* and finding their names and results quoted in the West. Kobzev was then the world expert on η-hyperhypersimple sets and had done marvels with them, but he missed Marchenkov's solution to Post's Problem. He recalled with a sad look that he could have done it very straightforwardly, putting together facts that he knew very well, if only he had thought of doing it. But he didn't.

When I published *Classical Recursion Theory*, I summed up in the Preface the set of events here described as follows: "I had learned that the Soviets were doing work that the Westerners did not know much about, and they themselves were largely unaware of what people did in the West. I found this an odd situation, and decided I would go to the Soviet Union to bridge the gap, at least in my knowledge."

At the end of my two years in the Soviet Union the bridge had indeed been built, at least in my knowledge. I cannot judge whether, or how much, my papers and books have influenced others, but it is a fact that towards the mid eighties a number of people, led by Richard Shore, started a fruitful work in areas of Recursion Theory that had previously been a Soviet monopoly. In my chapter for the *Handbook of Computability Theory* I have reported on the progress made on the problems I had stated in "Strong Reducibilities". And I hope that the relevant chapters of my book show how Recursion Theory must now be thought of as a collective Eastern and Western enterprise.

East and West have thus met, at least in our field, but this does not prove Kipling wrong. Indeed, it is too often forgotten that the negative, well know *incipit* quoted above was immediately corrected by the following positive verses, which can be taken as a poetic summary of the prosaic report in which I have indulged, and for which I beg forgiveness:

> *"But there is neither East nor West, Border, nor Breed, nor Birth,*
> *When two strong men [or schools] stand face to face, tho' they*
> *come from the ends of the Earth."*

Computability and Models

TRUTH-TABLE COMPLETE COMPUTABLY ENUMERABLE SETS

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Abstract We prove a truth-table completeness criterion for computably enumerable sets.

Keywords: Computably enumerable sets, truth-table reducibility, completeness criterion, fixed-point free functions

1. Introduction

There exist two kinds of completeness criteria for computably enumerable (c.e.) sets for several reducibilities. The first of them describes Turing complete c.e. sets by means of properties of productiveness of their complements, and the second describes Turing complete c.e. sets by means of diagonally noncomputable functions or, equivalently, fixed-point free functions, which are reducible to these sets. The author in his earlier papers [Ars81, Ars87, Ars89], see also [Soa87], worked out second kind criteria for Turing (T-), weak truth-table (wtt-) and m-reducibilities. For the latter the m-reducibility of a function f to a set A is defined as follows: $f \leq_m A$ if and only if there are computable functions a, b and g such that for all $x \in \omega$, $f(x) = a(x)$, if $g(x) \in A$, and $f(x) = b(x)$ otherwise. (The natural definition of m-reducibility of the function f to the set A via m-reducibility of the graph f to the set A is not suitable for this purpose, since if $\{\langle x, f(x)\rangle \mid x \in \omega\} \leq_m A$ and A is c.e., then the function f is computable, and by the recursion theorem has fixed points.)

The question of the existence of a second kind completeness criterion for truth-table (tt-) reducibility remained open. V.K. Bulitko in his paper [Bul91,

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