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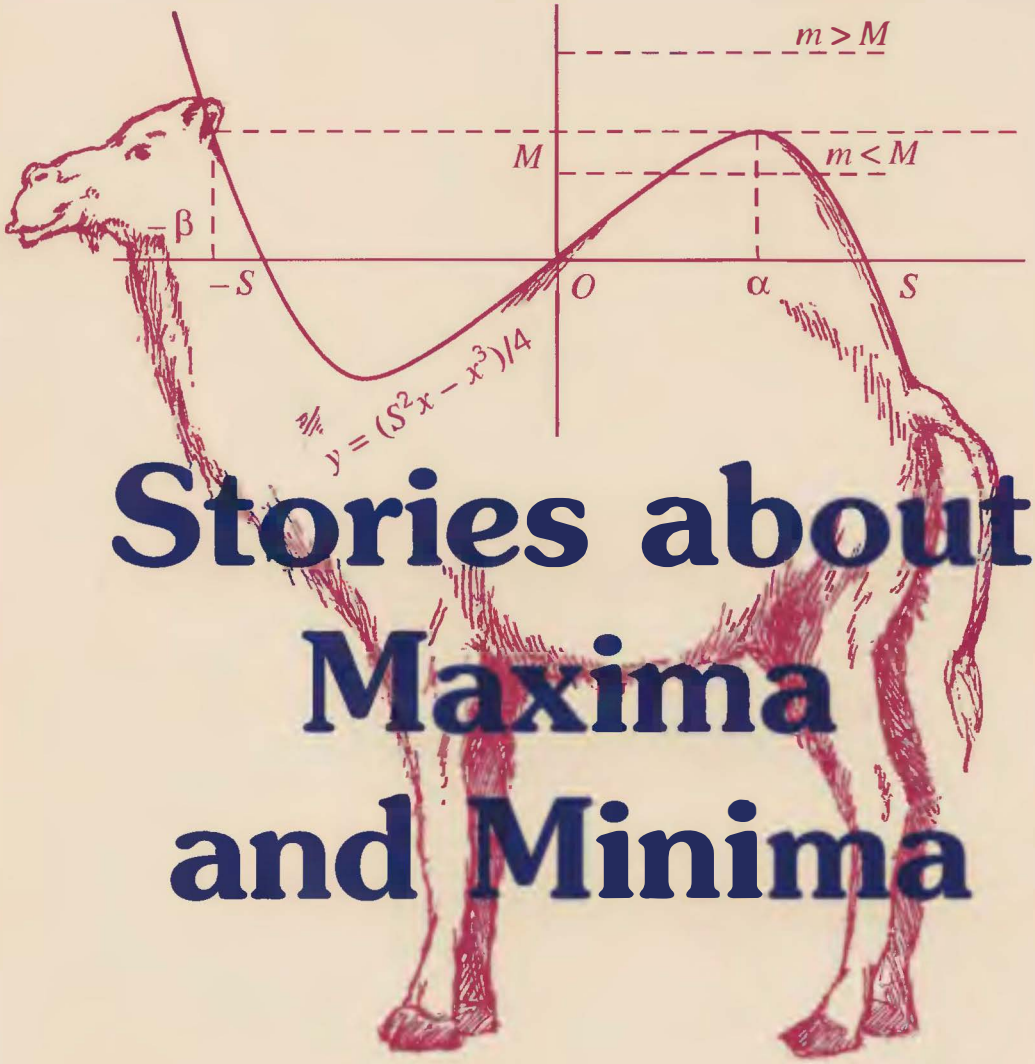

Stories about Maxima and Minima

V. M. Tikhomirov

American Mathematical Society
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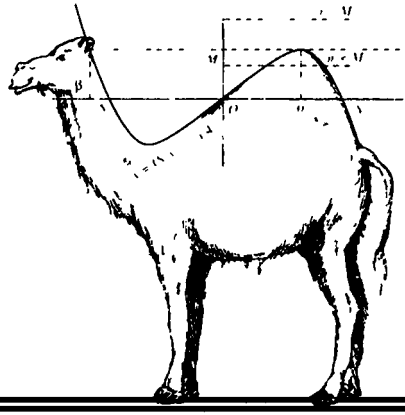

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В. М. ТИХОМИРОВ

РАССКАЗЫ О МАКСИМУМАХ И МИНИМУМАХ

«НАУКА», МОСКВА, 1986

Translated from the Russian by Abe Shenitzer

1991 *Mathematics Subject Classification.* Primary 00A07, 00A30, 00A35, 01-01, 46-01, 49-01, 49-03, 49J99

Library of Congress Cataloging-in-Publication Data

Tikhomirov, Vladimir M. (Vladimir Mikhaĭlovich), 1934-
Stories about maxima and minima/V. M. Tikhomirov
p. cm. (Mathematical world, ISSN 1055-9426, 1)
ISBN 0-8218-0165-1
1. Maxima and minima. 2. Calculus of variations. 3. Mathematical optimization.
QA306.T55 1990 90-21246
511'.66 dc20 CIP

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10 9 8 7 6 5 4 3 2 00 99 98 97 96 95

To the Memory
of My Dear Friend,
V. M. Alekseev

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Introduction

In daily life it is constantly necessary to choose the best possible (optimal) solution. A tremendous number of such problems arise in economics and in technology. In such cases it is frequently useful to resort to mathematics.

In mathematics, the study of maximum and minimum problems began a very long time ago, in fact, twenty-five centuries ago. For a long time there were no uniform ways of tackling problems for finding extrema. The first general methods of investigation and solution of extremal problems were created about 300 years ago, at the time of the formation of mathematical analysis.

Then it became clear that certain special optimization problems play a crucial role in the natural sciences. Specifically, it was found that many laws of nature can be derived from so-called "variational principles." According to these principles, given any collection of admissible motions, what distinguishes the actual motion of a mechanical system, or of light, electricity, a fluid, a gas, and so on, is that it maximizes or minimizes certain quantities. Some concrete extremal problems, whose content derives from the natural sciences (the brachistochrone problem, Newton's problem, and others), were posed at the end of the seventeenth century. The need to solve these, as well as many other problems of geometry, mechanics, and physics, led to the creation of a new branch of mathematical analysis that came to be known as the calculus of variations.

The intensive development of the calculus of variations continued for about two centuries. Many of the finest scientists of the eighteenth and nineteenth centuries took part in this process, and, by the beginning of this century, it seemed as if they had exhausted the topic.

But it turned out that this was not the case. The needs of practical life, especially in economics and technology, gave rise to new problems that could not be solved by the old methods. One had to advance. It was necessary to create a new field of mathematical analysis, known as "convex analysis," involving the study of convex functions and convex extremal problems.